Design Method for High Efficiency of Flow in Circular Pipes

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Abstract

Design engineers are often faced with the complex task of designing new collection systems. Methods based on Manning's equation are frequently used due to the availability of tables and graphs, which simplify the calculations. These methods lack accuracy except when laborious numerical methods are utilized. The design of a collection system seeks the computation of a diameter which produces an accepted velocity value without considering the water level in the computed pipe. Yet, the flow efficiency whether volumetric or circulation is an important design criteria. By considering the latter an increase in the volumetric capacity and circulation capacity of the flow in the pipe can be obtained. In this research, a new concept for the design of partially full pipe is proposed. It is based on Manning's equation and produces more efficient flow in pipe, i.e., the pipe is as fully exploited as possible.

Key words: pipes, steady uniform flow, pipe efficiency, Manning equation, pipe efficiency.

Introduction

The best design of sewer evacuation systems starts by studying the parameters which effect their operations, including technical, environmental and economical ones (McGhee 1991).
The flow in the collection system is usually considered uniform and steady. This type of flow has been investigated extensively by several reaserchers, where a number of approaches have been proposed including graphical methodes (Camp,1946; Chow, 1959; Swarna et al., 1990), semi-graphical sollutions (Zeghadnia et al., 2009), and nomogramms (McGhee, 1991) or tables (Chow, 1959). However, such approaches are usually considered limited and most of them are applicable only to limitted conditions. Numerical solutions are usually prefered in

practice, but these are difficult to apply and need to go through relatively lengthy trial and erros procedures.

A number of researchers have attempted to propose explicit equations for the computation of normal depht (Barr et al., 1986; Saatçi, 1990; Prabhata et al., 2004; Achour, B., and Bedjaoui, A., 2006.). Other authors prefer to simulate pressurized flow as free surface flow using the Preissmann slot method, hence, they can model the transition from free surface flow to surcharged state and vice versa (Cunge et al.,1980; Garcia Navarro et al.,1994; Capart et al., 1997; Ji, 1998; Trajkovic et al., 1999; and Ferreri et al., 2010).

The majority of research in this area is heavily focused on the determination of flow parameters, without looking at the performance of the flow inside the pipe. The concept of efficient pipe has not previously been explicitly discussed. The authors think that this is the first time this idea has been used in the direct calculation of pipes, which should draw the interest of researchers and designers alike. The efficiency of flow, therefore the efficiency of pipe is introduced as a measurable characteristic. Accordingly, the pipe will flow with maximum use of water surface, i.e., fully exploiting its surface area, while respecting the technical requirements, especially in terms of velocity.

In this paper we will shed some light on certain important technical considerations regarding the determination of hydraulic and geometrical parameters of partially filled pipes. The analysis takes into account other parameters like the slope, diameter, velocity, and pipe flow efficiency, using explicit solutions. Also, the limitations of the proposed solutions will be discussed.

Manning equation

Circular pipes are widely used for sanitary sewage and storm water collection systems. The design of sewer networks is generally based on the Manning model (Manning, 1891), where the flow section is mostly partially filled. The manning formula is commonly used in practice and is assumed to produce the best results when properly applied (Saatçi, 1990), (Lotfi, Z et al, 2014), (Zeghadnia,L et al, 2014). The usage of Manning model assumes the flow to be steady and uniform, where the slope, cross-sectional flow area, and velocity are not related to time, and are constant along the length of the pipe being analysed (Carlier, 1980). The Manning formula (Manning, 1891) used to model free surface flow can be written as follow:

$$Q = \frac{1}{n} R_h^{2/3} A S^{1/2} \tag{01}$$

or

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{02}$$

where:

Q: Flow rate (m^3/sec),

R_h*: Hydraulic radius (m),*

n: Pipe roughness coefficient (Manning n) (sec/m$^{1/3}$),

A: Cross sectional flow area (m^2),

S: Slope of pipe bottom, dimensionless,

V: Flow velocity (m/sec),

Eq. (1) and Eq.(2) can be written as functions of water surface angle shown in figure 01 as follow:

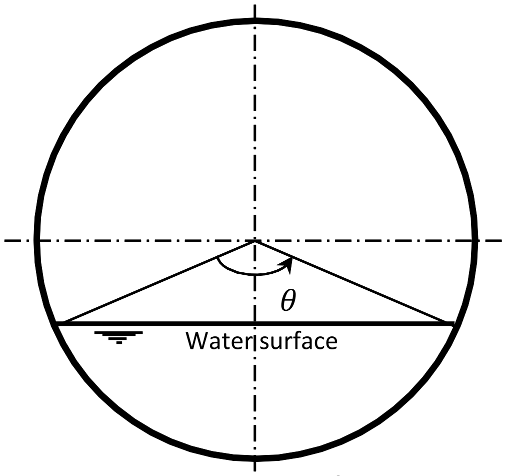

Figure 01 : Water surface angle

$$Q=\frac{1}{n}\left(\frac{D^8}{2^{13}}\right)^{1/3}\left[\frac{(\theta\text{-}\sin\theta)^5}{\theta^2}\right]^{1/3} s^{1/2} \tag{03}$$

$$V=\frac{1}{n}\left(\frac{D}{4}\right)^{2/3}\left[\frac{(\theta-\sin\theta)}{\theta}\right]^{2/3} s^{1/2} \tag{04}$$

$$A=\frac{D^2}{8}\left(\theta-Sin(\theta)\right) \tag{05}$$

$$P=\theta\frac{D}{2} \tag{06}$$

$$R_h=\frac{A}{P}=\frac{D}{4}\left(1-\frac{\sin(\theta)}{\theta}\right) \tag{07}$$

where:

D : Pipe diameter (m)

r : Pipe radius, $r=\frac{D}{2}$ *(m) ,*

P : Wetted perimeter(m)

θ *: Water surface angle (Radian).*

Eq. (03) and (04) for known values of flow *Q*, roughness *n*, slope *S*, and diameter *D*, can be solved only after a series of long iterations (Giroud et al., 2000). Eq.(04) can be substituted by Eq. (08) (Zeghadnia et al., 2009):

$$V=a\theta^{-2/5} \tag{08}$$

Where:

$$a=\frac{1}{n}\left(\frac{D}{4}\right)^{2/3} K^{2/3}s^{1/2} \tag{08-a}$$

$$K=\left[\left(\frac{nQ}{s^{1/2}}\right)^3\left(\frac{2^{13}}{D^8}\right)\right]^{1/5} \tag{08-b}$$

Therefore:

$$V=\left(\left(\frac{s^{1/2}}{n}\right)^3\left(\frac{2Q}{D}\right)^2\right)^{1/5}\theta^{-2/5} \tag{09}$$

Eq. (05) and (07) take the new forms as follows:

$$A = \left(\frac{D}{2}\right)^{2/5} \left(\frac{nQ}{s^{1/2}}\right)^{3/5} \theta^{2/5} \tag{10}$$

$$R_h = \left(\frac{2\,n\,Q}{D\,s^{1/2}}\right)^{3/5} \theta^{-3/5} \tag{11}$$

Estimation of volumetric or circulation efficiency

In order to simplify the computation, the calculation of pipe diameter is done frequently with the assumption that the pipe is flowing just full (under atmospheric pressure). Either flow or flow velocity can have maximum values, which correspond to certain water level in the pipe (Camp, 1944 and 1946). Below or above this level, the flow or the velocity values decrease, which means that the pipe is not flowing with its maximum efficiency. For best hydraulic design of sanitary sewage and storm water collection systems, it is not enough to determine the diameter which produces an acceptable flow velocity; but it is also necessary to determine the best diameter which allows higher efficiency and ensure that the pipe is fully exploited. To estimate the volumetric efficiency in pipe, we propose the flowing formula:

$$\mathrm{Q_{ef}} = 100\% * \left(1 - \frac{\mathrm{Abs(Q_{max} - q_r)}}{\mathrm{Q_{max}}}\right) \tag{12}$$

Where:

$\mathrm{Q_{ef}}$: Volumetric efficiency (%);

$\mathrm{Q_{max}}$: Maximum flow (m^3/sec);

$\mathrm{q_r}$: Flow in pipe (m^3/sec),

And, to compute the circulation efficiency in pipe, we propose the flowing formula:

$$\mathrm{V_{ef}} = 100\% * \left(1 - \frac{\mathrm{Abs(V_{max} - V_r)}}{\mathrm{V_{max}}}\right) \tag{13}$$

Where;

$\mathrm{V_{ef}}$: Circulation efficiency (%);

$\mathrm{V_{max}}$: Maximum velocity (m^2/sec);

$\mathrm{V_r}$: Velocity in pipe (m^2/sec), The volumetric and circulation efficiencies can be better explained using the graphical representation shown in the figure (02).

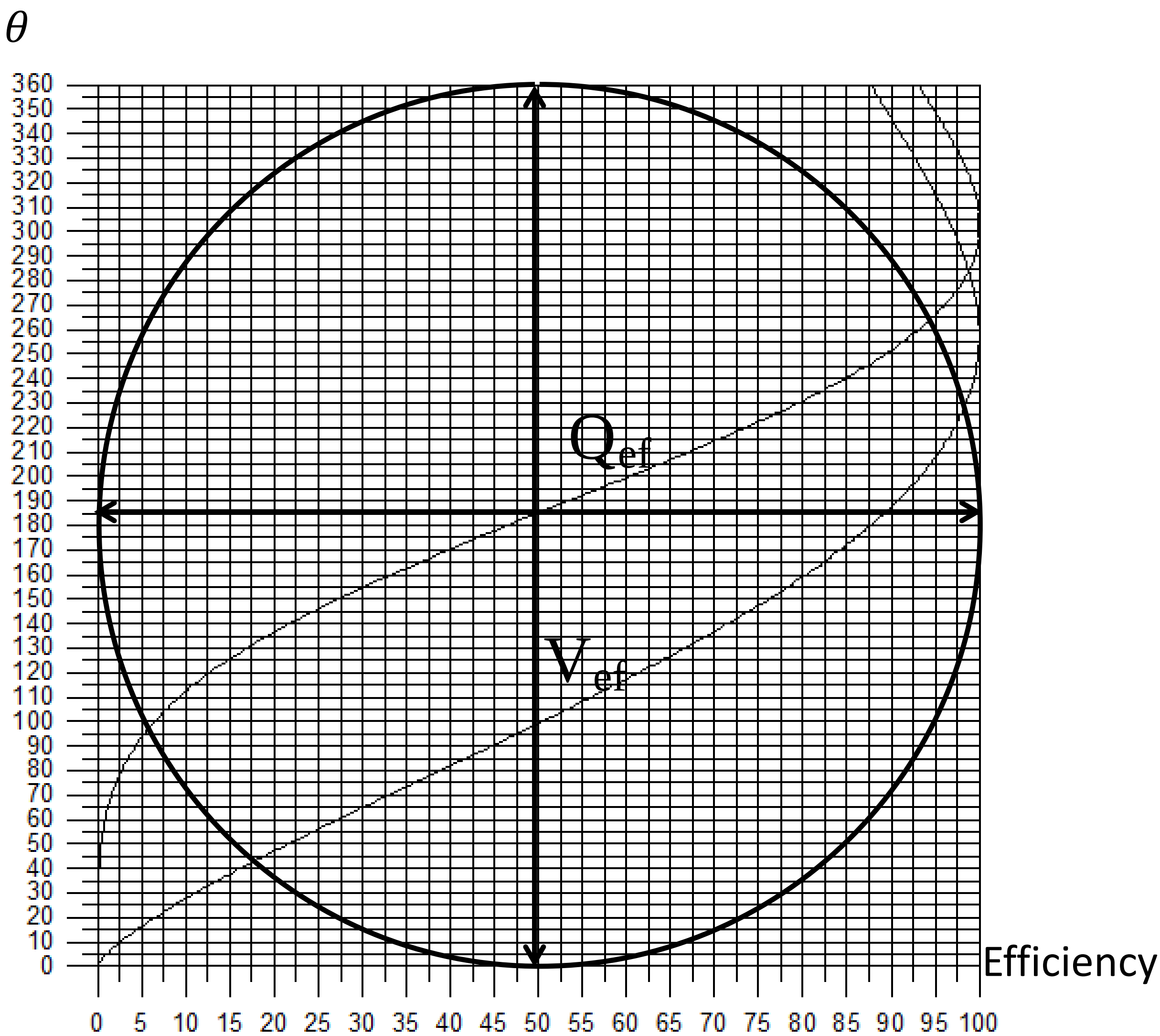

Figure02: Volumetric and circulation efficiency in circular pipe

Figure 02 shows that the volumetric or circulation efficiency depends on the level of filling of the pipe, and they do not vary in the same manner.

For $0° \leq \theta \leq 40°$, the volumetric efficiency is practically zero, while for $40° \leq \theta \leq 180°$, it is less than 50%. For $\theta = 185°$, the efficiency equals 50%, and it reaches its maximum value,

$Q_{ef} \cong 100\%$, at $\theta = 308°$. For $308° \leq \theta \leq 360°$ the volumetric efficiency decreases to reach a value of 93.09%.

On the other hand the variation of the circulation efficiency is more rapid than the volumetric efficiency. For $0° \leq \theta \leq 40°$ the circulation efficiency can reache 20%, and for $40° \leq \theta \leq 180°$ the efficiency reaches 85%. The circulation efficiency reaches its maximum value, $V_{ef} \cong 100\%$, at $\theta = 257°$. For $257° \leq \theta \leq 360°$ the circulation efficiency decreases to reach a value of 87.74%. Table 01 presents more details on the variation of both efficiencies as functions of θ.

Table01: The Volumetric and the circulation efficiency as function of water surface angle.

Water surface angle $\boldsymbol{\theta}$	**Volumetric efficiency Qef**	**Circulation efficiency Vef**
0	0	0
1	0	0.1201868
2	0	0.3028452
3	0	0.519973
4	7.629395E-006	0.7630229
5	1.525879E-005	1.027328
10	0.0003890991	2.586734
15	0.002235413	4.43598
20	0.007720947	6.498361
25	0.02018738	8.730227
30	0.04416656	11.10167
45	0.248764	18.84584
60	0.8313828	27.21751
90	4.296463	44.64105
120	12.72436	61.42837
150	27.19239	76.12539

180	46.46733	87.68739
210	67.11037	95.50261
240	84.79923	99.41671
251	89.81307	99.92023
252	90.22173	99.94276
253	90.62227	99.96152
254	91.01462	99.97654
255	91.39877	99.98783
256	91.77465	99.99545
257	92.14222	99.99939
258	92.50146	99.99969
259	92.85233	99.99638
260	93.19479	99.9895
293	99.79131	97.96639
294	99.85471	97.85783
295	99.91077	97.74696
296	99.95956	97.63379
297	99.99883	97.51839
298	99.96432	97.40081
299	99.93681	97.28107
308	99.98842	96.11406
347	95.35153	89.90553
360	93.0919	87.74671

Example

In this example we calculate the volumetric and circulation efficiencies for pipes with velocity $V_r = 0.88 m/sec$, q_r =0.15m^3/s, in a 500 mm pipe diameter, Qmax= 0.256 m^3/sec, Qfull= 0.238m^3/s, Vfull=1.212 m/sec, Vmax=1.30 m/sec.

Using equations (12) and (13), we find that Q_{ef} = 58.59% and V_{ef} = 67.68%. Hence this pipe is not efficient enough both in terms of volume and circulation. In this example, although the velocity is technically acceptable, this pipe is not flowing efficiently. Hence we need to find a better solution to insure high efficiency of the pipe, which will be discussed in the following sections.

Maximum volumetric efficiency

The efficiency is discussed in the following paragraphs in terms of pipe volume occupancy. The higher the latter, the more efficient the pipe is.

Maximum Flow condition

When cross sectional flow area A increases, it reaches its maximum value "A_{max}" with maximum volumetric efficiency at $\theta = 308.3236$, (Zeghadnia et al., 2009). From Eq. (03):

$$Q_{max} = 0.3349288 \frac{D^{8/3} S^{1/2}}{n} \tag{14}$$

For a pipe flowing full, the flow "Q" is expressed as follow:

$$Q_p = 0.3117909 \frac{D^{8/3} S^{1/2}}{n} \tag{15}$$

When we combine Eq. (14) and (15) we obtain the following:

$$Q_{max} = 1.06779512 Q_p \tag{16}$$

Eq. (16) presents the relationship between the flow for filled pipe and the maximum flow which, for any section, is possible only if the following condition is achieved (Carlier, 1980):

$$3\mathrm{PdA} - \mathrm{AdP} = 0 \tag{17}$$

Where (P is the wetted perimeter):

$$\mathrm{P} = \theta \mathrm{r} \Rightarrow \mathrm{dP} = \mathrm{r} \mathrm{d}\theta \tag{18}$$

$$\mathrm{A} = \frac{\mathrm{r}^2}{2}(\theta - \sin\theta) \Rightarrow \mathrm{dA} = \frac{\mathrm{r}^2}{2}(1 - \cos\theta)\mathrm{d}\theta \tag{19}$$

If we substitute the wetted perimeter "P", cross sectional flow area "A" and their derivatives in Eq. (17), we obtain the following:

$$3\frac{dA}{A} = \frac{dP}{P} \Rightarrow A^3 = P \tag{20}$$

If we combine Eq. (07) and (20), then Eq. (1) becomes:

$$Q = \frac{S^{1/2}}{n}\frac{A^{5/3}}{P^{2/3}} = \frac{S^{1/2}}{n}p^{-1/9} \tag{21}$$

From Eq. (21), the wetted perimeter can be rewritten as follow:

$$P = \left(\frac{S^{1/2}}{nQ}\right)^9 \tag{22}$$

By combining Eq. (06) and (22) we obtain the following:

$$D = \frac{2}{\theta_{Qmax}}\left(\frac{S^{1/2}}{nQ}\right)^9 \tag{23}$$

Eq. (23) can also be rewritten as follow:

$$D = 0.372\left(\frac{S^{1/2}}{nQ}\right)^9 \tag{24}$$

The use of Eq. (24) to compute the diameter, for flow max, is simple and direct when the roughness *n* and the slope *S* are known.

In the case where the slope *S* is unknown, Eq. (25) gives an explicit solution, if the flow *Q*, roughness *n* and diameter *D* are known.

$$S = \left(n\ Q\left(\frac{D}{0.372}\right)^{1/9}\right)^2 \tag{25}$$

Flow velocity limits

By combining Eq. (02), Eq. (07) and Eq. (20) we obtain:

$$V = \frac{S^{1/2}}{n}P^{-4/9} \tag{26}$$

If we substitute the wetted perimeter expression given in Eq. (22), into Eq. (26), we obtain the following:

$$V = \frac{s^{1/2}}{n}\left(\left(\frac{s^{1/2}}{nQ}\right)^9\right)^{-4/9} = \left(\frac{n}{s^{1/2}}\right)^3 Q^4 \tag{27}$$

The combination between Eq. (24) and (27) produces:

$$V = \frac{s^{1/2}}{n}\left(\frac{0.372}{D}\right)^{4/9} \tag{28}$$

From Eq. (27), the cross sectional area A can be rewritten as follow:

$$A = {}^{Q}/_{V} = \left(\frac{s^{1/2}}{n}\right)^3 Q^{-3} = R_R^3 Q^{-3} \tag{29}$$

We call " R_R " the resistance rate, which can be computed using equation (27) or (28) for maximum and minimum values of the flow velocity, respectively. Eq. (27) and Eq. (28) are applied only for the range of values given in table 02 and 03 in which the flow velocity varies between $0.5\mathrm{m/s} \leq \mathrm{V} \leq 5\mathrm{m/s}$ (Marc et al., 2006). In practice, the pipe diameters ranges generally between: $10\mathrm{mm} \leq \mathrm{D} \leq 2100\mathrm{mm}$.

Table 02. Flow velocity limits as a function of diameter and flow for the minimum value of $R_R = 0.4$, and $10\mathrm{mm} \leq \mathrm{D} \leq 250\mathrm{mm}$

D (mm)	Q (m3/s)	V(Q) m/s	V(D) m/s
10	0,60	2,00	2,00
12	0,59	1,84	1,84
16	0,57	1,62	1,62
20	0,55	1,47	1,47
25	0,54	1,33	1,33
32	0,53	1,19	1,19
40	0,51	1,08	1,08
50	0,50	0,98	0,98
63	0,49	0,88	0,88
75	0,48	0,81	0,82
90	0,47	0,75	0,75
100	0,46	0,72	0,72

110	0,46	0,69	0,69
125	0,45	0,65	0,65
140	0,45	0,62	0,62
160	0,44	0,58	0,58
200	0,43	0,53	0,53
225	0,42	0,50	0,50
250	0,42	0,50	0,50

Table 03. Flow velocity limits as a function of diameter and flow for the maximum value of R_R =1, and $10\text{mm} \leq D \leq 250\text{mm}$

D (mm)	Q (m3/s)	V(Q) m/s	V(D) m/s
10	1,49	4,99	4,99
12	1,46	4,60	4,60
16	1,42	4,05	4,05
20	1,38	3,67	3,67
25	1,35	3,32	3,32
32	1,31	2,97	2,98
40	1,28	2,69	2,69
50	1,25	2,44	2,44
63	1,22	2,20	2,20
75	1,19	2,04	2,04
90	1,17	1,88	1,88
100	1,16	1,79	1,79
110	1,14	1,72	1,72
125	1,13	1,62	1,62
140	1,11	1,54	1,54
160	1,10	1,45	1,45
200	1,07	1,32	1,32
225	1,06	1,25	1,25
250	1,05	1,19	1,19

Tables 02 and 03 presents the solutions for Eqs (27) and (28). By comparing the flow velocities in Table 02 and 03 we can conclude that the resistance rate R_R influences remarkably these values. For diameters that vary in range between $10mm \leq D \leq 250mm$, the minimal value of R_R should not be lower than 0.4. This yields a variation in the flow in the range given by the following relationship:

$$0.42\, {m^3}/{s} \leq Q \leq 0.6\, {m^3}/{s}. \tag{29-a}$$

The same diameter range accepts another boundary as maximum flow value for R_R =1. This generates the following flow values range:

$$1.05\, {m^3}/{s} \leq Q \leq 1.49\, {m^3}/{s}. \tag{29-b}$$

If we expand the range of variation in diameter: $315mm \leq D \leq 2100mm$, while we keep the condition of flow velocity as indicated above, we obtain the following results given in Tables 04 and 05. The latter present the variation of flow values as a function of the diameter and the limit values of R_R. We can summarize the variation of flow according to the variation of R_R as follow:

- For the minimum value of R_R =1.05, the flow varies, according to table 04 results, as follow:

$$0.87\, {m^3}/{s} \leq Q \leq 1.07\, {m^3}/{s} \tag{29-c}$$

- For the maximum value of R_R =4.64, the flow varies, according to table 05 results, as follow:

$$3.83\, {m^3}/{s} \leq Q \leq 4.73\, {m^3}/{s} \tag{29-d}$$

Other results could easily be obtained using different values of R_R within its accepted limits.

Table 04. Flow velocity limits as function of diameter and flow for minimum R_{R}(min) =1.05, $315mm \leq D \leq 2100mm.$

D (mm)	Q (m3/s)	V(Q) m/s	V(D) m/s
315	1,07	1,13	1,13
400	1,04	1,02	1,02
500	1,02	0,92	0,92
600	1,00	0,85	0,85
700	0,98	0,79	0,79
800	0,96	0,75	0,75
900	0,95	0,71	0,71
1000	0,94	0,68	0,68
1100	0,93	0,65	0,65
1200	0,92	0,62	0,62
1300	0,91	0,60	0,60
1400	0,91	0,58	0,58
1500	0,90	0,56	0,57
1600	0,89	0,55	0,55
1700	0,89	0,53	0,53
1800	0,88	0,52	0,52
1900	0,88	0,51	0,51
2000	0,87	0,50	0,50
2100	0,87	0,50	0,50

Table 05. Flow velocity limits as function of diameter and flow for maximum R_{R}(max) =4.64, $315mm \leq D \leq 2100mm$

D (mm)	Q (m3/s)	V(Q) m/s	V(D) m/s
315	4,73	5,00	5,00
400	4,60	4,49	4,49
500	4,49	4,07	4,07
600	4,40	3,75	3,75
700	4,33	3,50	3,50

800	4,26	3,30	3,30
900	4,21	3,13	3,13
1000	4,16	2,99	2,99
1100	4,11	2,87	2,87
1200	4,07	2,76	2,76
1300	4,04	2,66	2,66
1400	4,00	2,57	2,57
1500	3,97	2,50	2,50
1600	3,95	2,43	2,43
1700	3,92	2,36	2,36
1800	3,89	2,30	2,30
1900	3,87	2,25	2,25
2000	3,85	2,20	2,20
2100	3,83	2,15	2,15

Maximum circulation efficiency

In this section the efficiency of the pipe is treated based on the circulation of flow. We look at the variation of the circulation efficiency from different levels. Then we will present how to obtain the maximum exploitation of the pipe.

Condition of maximum Flow velocity

Flow under condition of maximum flow velocity is an important in sewage network drainage. In these types of flow condition it is imperative to check the following condition (Carlier, 1980):

$$\mathrm{PdA} - \mathrm{AdP} = 0 \qquad (30)$$

Where:

P : Wetted perimeter (m)

A : Cross sectional flow area (m^2)

The combination between the Eq. (18), Eq. (19) and Eq. (30) gives the following:

$$-\theta\cos\theta + \sin\theta = 0 \tag{31}$$

Eq. (31) can be solved iteratively. The use of the Bisection Method (Andre, 1995) gives the following results (where the absolute error equal to 10^{-6}): $\theta = 257{,}584$

$$\frac{dA}{A} = \frac{dP}{P} \tag{32}$$

From Eq. (06), Eq. (10) and (32) and after many simplifications we obtain the following formula:

$$D = \frac{0.445\, n\, Q}{S^{1/2}} \tag{33}$$

Therefore, Eq. (10) can be rewritten as follow:

$$A = \frac{n\, Q}{S^{1/2}} \tag{34}$$

Eq. (33) for known flow Q, roughness n, and slope S, gives explicit solution for the diameter. The slope S can be also calculated directly by Eq. (35) if the flow Q, roughness n and diameter D are known parameters:

$$S = \left(\frac{2\, n\, Q}{4.49\, D}\right)^2 \tag{35}$$

According to Eq. (34), it is easy to deduce that the flow velocity is equal to the ratio of square root of the slope and roughness as follow:

$$V = \frac{S^{1/2}}{n} = \frac{0.445 Q}{D} \tag{36}$$

From Eq. (36), and at first glance we can conclude that the flow velocity depends only on the slope and roughness. This is true in this case. However, this conclusion must be related to another reality, that this formula is conditioned by the fullness degree in the pipe, which means the diameter used in Eq. (36) should be computed using Eq.(33) firstly.

Recommended limits

The proposed model of flow under condition of maximum velocity is governed by flow velocity limits, which produce a succession of limits of the other parameters: flow, slope and pipe roughness for the range of values presented in table 06 and 07:

Table 06. Recommended limits of flow velocity as a function of diameter and flow for: $R_R(\text{min}) = 0.5$, and $10mm \leq D \leq 2100mm$.

Table 07. Recommended limits of flow velocity as a function of diameter and flow for: $R_R(\text{max}) = 5$; and $10mm \leq D \leq 2100mm$.

From the parameters values shown in tables 06 and 07, we can easily conclude that the resistance rate R_R is an important parameter, where it can allow for the enlargement or the narrowing of the range of validity. In the case of maximum velocity the equations of applicability can be presented as follow:

1. For minimal value of $R_R = 0.5$ and for diameters range of $10\text{mm} \leq \text{D} \leq 2100\text{mm}$, the flow varies as follow :

$$0.01 \, {m^3}/{s} \leq Q \leq 2.36 \, {m^3}/{s} \tag{36-a}$$

2. If $R_R = 5$ and $10\text{mm} \leq \text{D} \geq 2100\text{mm}$, the flow varies as follow:

$$0.11 \, {m^3}/{s} \leq Q \leq 23.60 \, {m^3}/{s} \tag{36-b}$$

From the above, and in a similar way to the case of flow under condition of maximum velocity or maximum flow, it’s imperative to respect the variation of the resistance rate R_R which gives afterwards acceptable values for flow velocity, and not necessary desired flow, because each range of R_R generates different range of flow. The range of flow values are given as follows:

a) Case of flow max:

$$Q_{D=250mm} \leq Q_{Known} \leq Q_{D=10mm} \tag{36-c}$$

Or :

$$Q_{D=2100mm} \leq Q_{known} \leq Q_{D=315mm} \tag{36-d}$$

b) Case of velocity max:

$$Q_{D=10mm} \leq Q_{Known} \leq Q_{D=2100mm} \tag{36-e}$$

Let us take practical field scenarios through the following two examples.

Example 1

A pipe with manning coefficient $n = 0.013$, slope $= 0.02\%$, transport a flow of $1.05\ m^3/s$. Compute the pipe diameter for maximum volumetric efficiency

Solution

1. First we must check if the value of the resistance rate R_R is respected so we can use the model :

$$1.05 \leq R_R = \frac{S^{1/2}}{n} = 1.08 \leq 4.64$$

The resistance rate belongs to the allowable range. From table 03 and 04 we can conclude that diameter varies as follows:

$$315mm \leq D \leq 2100mm$$

2. Checking the flow range:

From Eq. (24) it is easy to compute $Q_{D=315mm}$ and $Q_{D=2100mm}$

$$Q_{D=315mm} = \left(\frac{0.372}{D}\right)^{1/9} \frac{i^{1/2}}{n} = 1.10\, m^3/s$$

$$Q_{D=2100mm} = \left(\frac{0.372}{D}\right)^{1/9} \frac{i^{1/2}}{n} = 0.89\, m^3/s$$

$$0.89\, {m^3}/{s} \leq Q = 1.05 \leq 1.10\, {m^3}/{s}$$

Q belongs to the allowable range.

3. From Eq. (24) the diameter is calculated as:

$$D = 0.372\left(\frac{i^{1/2}}{nQ}\right)^9 \cong 500mm$$

4. Checking of the flow velocity: From equation (27) we obtain the following:

$$V = 0.95\, m/s$$

The flow velocity value is acceptable; the same for the diameter, which will produce, with the other parameters, the maximum flow. (Which corresponded to fullness degree θ_{Qmax}).

Example 2

Let us to use the same data for the previous example to calculate the new diameter in case of maximum efficiency of flow circulation in pipe.

Solution

1. Checking for allowable R_R range :

$$0.5 \leq \frac{S^{1/2}}{n} = 1.08 \leq 5$$

Therefore, the diameter varies as follow:

$$10mm \leq D \leq 2100mm$$

2. Checking for the flow range :

   Eq. (33) allows the calculation of $Q_{D=10mm}$ and $Q_{D=2100mm}$

$$Q_{D=10mm} = \left(\frac{D}{0.445}\right) \quad \frac{i^{1/2}}{n} = 0.02\, m^3/s$$

$$Q_{D=2100mm} = \left(\frac{D}{0.445}\right) \quad \frac{i^{1/2}}{n} = 5.10\, m^3/s$$

$$0.02\, {m^3}/{s} \leq Q = 1.05 \leq 5.10\, {m^3}/{s}.$$

Hence, the flow is within the allowable range.

3. Computation of the pipe diameter

From Eq. (33) the pipe diameter equals to:

$$D = \frac{0.445\; n\; Q}{i^{1/2}} \cong 400mm$$

From the above, the pipe diameter *D* is a known parameter, the flow velocity depends only on the slope S and roughness n, and from equation (36) we obtain the following:

$$V = \frac{i^{1/2}}{n} = 1.08\; m/s$$

The flow velocity is within the acceptable range.

Conclusion

A new conception of the design of partially full flow in circular pipe is proposed using the new concept of volumetric and circulation efficiency. Two types of flow are considered: flow under condition of maximum flow, and flow under maximum velocity respectively. These are important criteria for the waste water evacuation. For both cases, direct and easy solutions have been elaborated to calculate the pipe diameter, flow velocity and slope. In the first the diameter and slope can be calculate with Eq. (24) and Eq. (25). For the second case Eq. (33) and (35) are recommended. For each case the computation of flow velocity is possible.

The limitation of the solution range has been discussed too. The proposed equations are elaborated to obtain high efficiency of flow in circular pipes, while meeting the technical requirements.

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Notation

Q: Flow rate in m3/s,

R_h: Hydraulic radius,

n: Pipe roughness coefficient (Manning n),

A: Cross sectional flow area,

S: Slope of pipe bottom, dimensionless,

V: Flow velocity m/s,

r : Pipe radius, let's : $r = \frac{D}{2}$,

D : Pipe diameter,

P : Wetted perimeter,

θ : Water surface angle,

Qef : Volumetric efficiency;

Q_{max}: Flow max;

q_r: Flow in pipe,

Vef : Circulation efficiency;

V_{max}: Velocity max;

V_r: Velocity in pipe,

A_{max}: Cross sectional area correspond to Q_{max},

Q_p : Flow in full section,

θ_{Qmax}: Water surface angle correspond to Q_{max}

R_R: the resistance rate.

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